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PROPAGATION OF BENDING-TORSIONAL WAVES IN THIN-WALLED BEAMS OF OPEN CROSS SECTION

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V. B. MESHCHERIAKOV
(Moscow)

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Dispersion equations of the phase and group velocities of the bending-torsional waves in thin-walled, open section beams, with the deformations due to shear taken into account, are given. It is shown that in the general case six modes of propagation of the bending-torsional waves exist. A numerical problem is considered for a channel section beam. The limiting frequencies are discussed, which divide the whole frequency range into regions with different numbers of possible modes of wave propagation.

A system of equations describing the bending-torsional oscillations of thin-walled, open section beams was obtained earlier in [1], and for the channel and I-section beams in [2]. However, in [1] the shear deformations which arise in bending-torsion were not taken into account. The equations in [2] were derived by approximate methods and this led to violation of symmetry in the matrix of the geometrical characteristics (the matrix S).

Using the analytic method developed in [3], we constructed in [4, 5] a general theory of equilibrium of motion of thin-walled, open profile beams, with the shear deformation taken into account. In the absence of external loads, the equations of bending-torsional motion can be written in the following matrix form [5]:

$$\mathbf{B}\boldsymbol{\vartheta}^{\mathrm{IV}} - \mathbf{B}_1\boldsymbol{\vartheta}'' - \frac{\rho}{E}\mathbf{B}\boldsymbol{\vartheta}^{\cdot\cdot''} + \rho F\mathbf{A}\boldsymbol{\vartheta}^{\cdot\cdot} + \frac{E}{G}\rho F\mathbf{A}\mathbf{S}\left(\boldsymbol{\vartheta}^{\cdot\cdot''} - \frac{\rho}{E}\boldsymbol{\vartheta}^{\cdot\cdot\cdot\cdot}\right) = 0 \tag{1}$$

$$\mathbf{B} = \operatorname{diag}(EI_x, EI_y, EI_\omega), \quad \mathbf{B}_1 = \operatorname{diag}(0, 0, GI_d)$$

$$\mathbf{A} = \begin{Vmatrix} 1 & 0 & -a_x \\ 0 & 1 & a_y \\ -a_x & a_y & r^2 \end{Vmatrix}, \quad \mathbf{S} = \begin{Vmatrix} S_{yy}/I_x & S_{yx}/I_x & S_{y\omega}/I_x \\ S_{xy}/I_y & S_{xx}/I_y & S_{x\omega}/I_y \\ S_{\omega y}/I_\omega & S_{\omega x}/I_\omega & S_{\omega\omega}/I_\omega \end{Vmatrix}$$

Here E and G are the moduli of elasticity, ρ is the material density, F is the area of transverse cross section, $\mathbf{B}$ and $\mathbf{B}_1$ are the rigidity matrices, I_x and I_y are the moments of inertia relative to the principal axes, I_ω is the principal sectorial moment of inertia, I_d is the torsional moment of inertia, $\boldsymbol{\vartheta}$ is the general displacement vector the components of which are the angles of rotation of the transverse cross sections relative to the principal axes (ϑ_x, ϑ_y) and to the warping of the cross sections ϑ_ω, $\mathbf{A}$ and $\mathbf{S}$ are the matrices of geometrical characteristics, while a_x and a_y are the coordinates of the principal sectorial pole in the cross section, r is the polar radius of inertia of the cross section and S_{ij} denote the geometrical characterisitics used to account for the effects of the shear deformation [3, 4]. In (1) primes denote differentiation with respect to the longitudinal coordinate z, and dots denote the differentiation with respect to time t. We note that the matrix S in [1] does not contain the elements $S_{\omega\omega}$, $S_{\omega x}$, $S_{\omega y}$, while in

[2] the condition of symmetry $S_{\omega x} = S_{x\omega}$ and $S_{\omega y} = S_{y\omega}$ is violated.

The displacements ξ_x, ξ_y of the beam axis and the angles of rotation of the cross sections ξ_ω about this axis, together form a vector ξ which is connected with the vector ϑ by the following matrix equation:

$$\xi' = -\vartheta - \frac{E}{G}\,\mathbf{S}\left(\vartheta'' - \frac{\rho}{E}\,\ddot{\vartheta}\right) \tag{2}$$

If we neglect the shear deformations, the system (1) and (2) yields the equations which correspond to the Vlasov theory [6].

Let us turn our attention to the problem of velocities of propagation of the bending-torsional waves in a thin-walled beam. We shall seek the solutions of the system (1) in the form

$$\vartheta(z, t) = \| A_j \exp\{i\,[(\nu z + \psi t) + \alpha_j]\}\|, \quad j = x, y, \omega \tag{3}$$

where A_j, α_j are constants, ν is the wave number and ψ is the wave frequency. Substituting (3) into (1), we arrive at a system of algebraic equations for A_j. To obtain a nontrivial solution, we equate the determinant to zero. This yields an equation connecting the wave number with the frequency

$$\det\left[\mathbf{B}\nu^4 + \mathbf{B}_1\nu^2 - \rho F\mathbf{A}\psi^2 - \frac{\rho}{E}\,\mathbf{B}\nu^2\psi^2 + \frac{E}{G}\,\rho F\mathbf{A}\mathbf{S}\left(\nu^2\psi^2 - \frac{\rho}{E}\,\psi^4\right)\right] = 0 \tag{4}$$

The phase velocities c and group velocities c_g of the waves are found from the relations

$$c = \psi / \nu, \quad c_g = d\psi / d\nu \tag{5}$$

The twelfth order equation (4) and relations (5) together yield six nonnegative values for the phase and group wave velocities, depending on the wave number (or on the frequency). In the general case of an arbitrary open section, the bending-torsional waves can propagate through the beam in six different ways.

We can write Eq. (4) in the form

$$\det\left[\left(\mathbf{B}\mathbf{K} - \frac{EF}{\psi^2}\,c_2^2\mathbf{A}\right)c^4 - \left(\mathbf{B}c_2^2 + \mathbf{B}\mathbf{K}c_1^2 - \frac{c_1^2c_2^2}{\psi^2}\,\mathbf{B}_1\right)c^2 + \mathbf{B}c_1^2c_2^2\right] = 0 \tag{6}$$

$$c_1 = \sqrt{E/\rho}, \quad c_2 = \sqrt{G/\rho}, \quad \mathbf{K} = -EF\mathbf{B}^{-1}\mathbf{A}\mathbf{S}$$

Let us consider the case in which the principal sectorial pole coincides with the center of gravity in the cross section. This occurs e. g. when the beam has two planes of symmetry. All matrices entering Eq. (6) become diagonal, and the equation itself splits into three independent equations. Consider one of these equations referring to the torsional waves

$$c^4\left(k_{\omega\omega}\frac{E}{G} - \frac{EFr^2}{\rho I_\omega \psi^2}\right) - c^2\left(1 + k_{\omega\omega}\frac{E}{G} - \frac{GI_d}{\rho I_\omega\psi^2}\right)c_1^2 + c_1^4 = 0 \tag{7}$$

Here $k_{\omega\omega}$ is the coefficient related to the form of the cross section under the bending torsion. Assuming that the coefficient accompanying c^4 is zero, we obtain the value of the limiting frequency

$$\psi_{*\omega} = \sqrt{\frac{GFr^2}{\rho I_\omega k_{\omega\omega}}} = \frac{c_2}{R_\omega}, \quad R_\omega = \sqrt{\frac{I_\omega k_{\omega\omega}}{Fr^2}} \tag{8}$$

When $\psi = \psi_{*\omega}$, Eq. (7) yields a single value for the phase velocity, when $\psi > \psi_{*\omega}$ another value appears and we have two modes of propagation of the torsional waves. When $\psi < \psi_{*\omega}$, the second root of (7) is imaginary, hence no waves of the second type are transmitted.

The limiting frequency $\psi_{*\omega}$, defined by (8) can be given the following physical explanation. The quantity c_2 represents the velocity of propagation of the shear wave and the quantity R_ω can be called the reduced radius of inertia of the cross section. Then the quantity $\psi_{*\omega}$ is inversely proportional to time in which the shear wave transverses a distance equal to the reduced radius of inertia of the cross section.

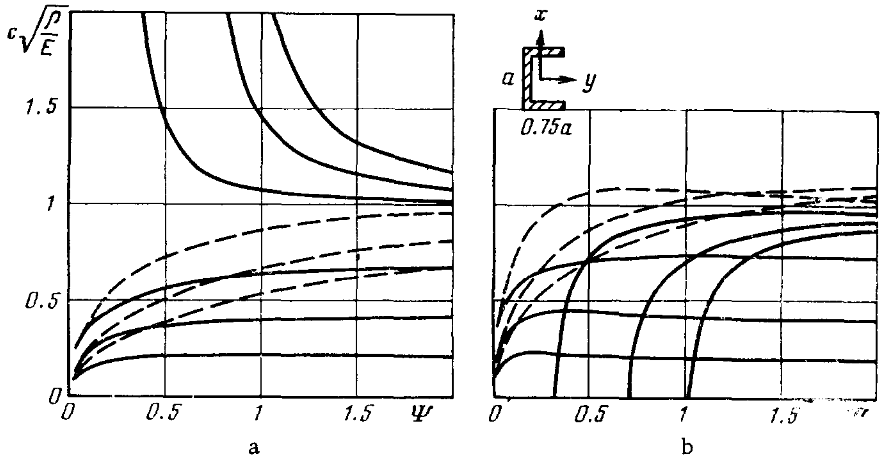

Fig. 1

Similar results can be obtained for the flexural waves. The values of the limiting frequencies are obtained from the formulas

$$\psi_{*x} = \sqrt{\frac{GF}{\rho I_x k_{yy}}}, \quad \psi_{*y} = \sqrt{\frac{GF}{\rho I_y k_{xx}}}$$

In a beam with an arbitrary open transverse cross section, the values of the limiting frequencies can be found from the matrix equation

$$\det\left[\mathbf{BK} - \frac{EF}{\psi^2} c_2^2 \mathbf{A}\right] = 0$$

As an example, we consider a beam with channel cross section. Figure 1 shows the phase velocities (a) and group velocities (b) of the bending and bending-torsional waves versus frequency

$$\Psi = \psi \sqrt{\rho I_y / EF}$$

The dashed lines depict the results obtained from the Vlasov theory. The ratio of the moduli of elasticity was assumed to be $E / G = 2.6$ and a numerical algorithm was used on a computer to perform the calculations.

We can see from the graphs that the limiting values of the frequencies divide the whole 0 to ∞ frequency range into four regions with different numbers of possible modes of wave propagation. In the low frequency region only three modes of wave propagation are possible; in the present case these are represented by a bending wave (in the plane of symmetry) and two coupled bending-torsional waves. As the frequency increases, the number of possible modes increases by one on each passage through the limiting frequency to reach six possible modes in the last region.

The wave group velocities given by (6) do not exceed those of the expansion waves.

The propagation velocities of the high frequency waves approach those of the Rayleigh surface waves. For the case of the bending waves the result is well known, and is in good agreement with the exact solution of the theory of elasticity [7].

In the high frequency region the group velocities of the lower wave forms are practically constant. This makes it possible to neglect the wave dispersion when estimating the contribution of the higher frequencies to the acceleration appearing within the beam under the action of short duration loads [8].

It should be noted that the results obtained using the Vlasov theory do not contain additional modes of wave propagation. The velocity values obtained from the lower modes are too high and do not match the real process. The group wave velocities exceed, in a certain frequency range, the speed of sound in the material.

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